# LMMSE Demosaicing for multicolor CFAs 

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#### Abstract

Digital cameras overlay a Color Filter Array (CFA) over the sensor which sub-samples the color information of the scene. The full color image is then recovered using a class of algorithms known as demosaicing. The color filters used for CFA are predominantly Red, Green and Blue. In this article we propose an algorithm based on Linear Minimum Mean Square Error (LMMSE) which can demosaic images from any color (linear combination of Red, Green and Blue) filter arranged in any order in the CFA. We also propose optimum CFAs based on combination of RGB colors which outperform the state of art CFAs in image reproduction with less computational complexity.


## Introduction

We need to add light intensities of three color primaries at each pixel to produce color images. To capture this information of three colors simultaneously requires a three sensor system which is expensive. Therefore, today digital cameras overlay a Color Filter Array (CFA) on the sensor to sub-sample a scene into a mosaic of colors. This mosaiced image is then processed to recover the full color image, known as demosaicing. Red, green and blue are the colors used predominantly in these filters. It corresponds qualitatively to the phosphors (CRTs) or color filters (LCDs) used in the display monitors and therefore well suited.

The classic Bayer CFA [7] is ubiquitous in todays digital cameras. However it has its limitations as common demosaicing algorithms give false colors and artifacts due to periodic nature of arrangement. This is compensated by using increasingly complex algorithms like edge preservation ones which demosaic along a contour and not across it $[12,15,17,18,22-24,33,39,40]$. However this has its own computational cost, which usually forces designing specific processors for embedded real time demosaicing.

In digital cameras our choice of color filters is not limited to RGB and in this paper we try to identity which color filter choices and their arrangement can give us the best image reproduction using computationally simple algorithms.

In the late 1990s and early 2000s cameras did make use of Cyan, Yellow, Magenta or even Emerald colors in the CFAs. Kodak is known for using CYYM [7], Canon and Nikon for CYGM [35] and Sony for RGBE [36] color filters in their cameras (Figure 1). Recently, white (absence of any color filter) has also made its appearance as it theoretically helps to recover more dynamic range and therefore has applications in low light photography. Lately there has been a renewal in designing optimal CFAs considering more than three primary (RGB) colors. For the purpose of this paper we limit ourselves to considering new color filters as linear combination of RGB color filters as in several other studies $[6,8,14,16,38]$. We term these CFAs as multicolor CFAs.

If we perform a two dimensional spatial DFT (Discrete


Figure 1. CFAs with RGB and multicolor CFAs $[7,35,36]$

Fourier Transform) of a CFA image we see that the luminance and chrominance components are heavily multiplexed, however for periodic CFAs like the Bayer they are localized separately $[2,3]$. Luminance in the low frequency regions (centered at $(0,0)$ ) and chrominance in the higher. To demosaic successfully, we need to design filters to separate them as cleanly as possible. The chrominance component is sub-sampled so we need to interpolate it and add back the luminance to get the final color image. The design philosophy for CFAs is to move the chrominance components away from the horizontal and vertical axis where luminance has its maximum intensity $[6,8,9,11,14,16]$. See Figure 2 for average of two dimensional DFT for CFA images simulated on Kodak database images [1] for several different CFAs. We see that CFAs like Yamanaka [29], Holladay [26], Bai [6], Hao 40 and Hao 50 (to some extent) [38] fulfill this criteria. Then we need to design demosaicing low pass and high pass filters which can separate luminance and chrominance. Generally this approach works for periodic CFA patterns but it cannot work for random CFAs as chrominance components are present across the entire frequency band.

For random or periodic CFAs we can consider demosaicing as an inverse problem of estimating the missing colors from the sampled ones. Let us consider the solution to be a black box with input as the CFA image and output as the full color image. If we have a database of true color images we can simulate a CFA image, pass it through black box to get a reconstructed image. The goal of designing the black box is to minimize the difference between the original image and the reconstructed image. We can select from a family of linear [19, 20, 30-32] or non linear solutions $[10,21]$ to design our black box. The problem with such black box models is that the number of outputs should be less than the inputs to ensure that the problem is overdetermined. However this is not the case for the demosaicing problem, therefore the solution is not stable. To overcome this limitation we introduce the notion of neighborhood to increase the inputs. A non linear solution could work better than a linear one as it allows greater degree of freedom in designing the black box, however it could overfit. Therefore we choose a linear solution for our model as it is very fast and allows us to quickly optimize the CFA color selection. In the next section we explain the matrix model of our Linear Minimum Mean Square Error (LMMSE) based solution. Also then we find the optimal CFA pattern which gives the best performance by


Figure 2. Average two dimensional spatial DFT (Discrete Fourier Transform) for all CFA images constructed from the Kodak database. Each sub-image corresponds to a CFA, ( $f x=0, f y=0$ ) corresponds to the center of sub-image. In blue are low values and red are high values. Sub-images (a) Bayer [7], (b) Yamanaka [29], (c) Holladay [26] (d) Bai [6], (e) Hao40 [38], (f) Hao50 [38]
solving a constrained minimization problem.

## Model for Demosaicing for multicolor CFAs

Let us consider $Y$ to be a RGB image of size $H$ rows, $W$ columns and $P=3$ color channels. $X$ is the corresponding CFA image of size $H$ rows and $W$ columns. For modeling we convert these images into column vectors such that the demosaicing problem can be expressed as matrix multiplication. This helps in expressing the problem in linear terms. Also our choice of algorithm is block shift invariant, the same operation is repeated for each basis pattern of size $h$ rows and $w$ columns. For e.g., in Bayer CFA $h=w=2$. We unfold $Y$ as column vectors $y$ of size $P h w \times H W /(h w)$ and $X$ as column vectors $x$ of size $h w \times H W /(h w)$, unfolding each basis pattern as single column. Any of our new color can be considered as a linear combination of three primaries red, green and blue. So let us define $C$ to be an artificial color image, such that $C_{i}=\alpha_{i} R+\beta_{i} G+\gamma_{i} B$. We can consider as many colors $P_{C}$ as the size of basis pattern $h \times w$, i.e. $P_{C} \leq h w$. Here white is a case where $\alpha=\beta=\gamma=1$. We could also consider white to be $\alpha=\beta=\gamma=1 / 3$, however ours being a linear model, the demosaicing operator we will define later is invariant to this scaling. We can now express $c$ of size $P_{C} h w \times H W /(h w)$ which is unfolded image $C$ of size $H W P_{C}$ as:

$$
\begin{equation*}
c=A y \tag{1}
\end{equation*}
$$

where $A$ is matrix of $\left[\alpha_{i} \beta_{i} \gamma_{i}\right.$ ] for each element in $y$, of size $P_{C} h w \times$ Phw. For example for a RGBW CFA as shown in Figure 1, the matrix $A$ will be

$$
A=\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \otimes\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 1 & 1
\end{array}\right]
$$

where $\otimes$ is the kron operator
Now $x$ is the CFA image in multi colors which is $M$ projection of c. $M$ is $h w \times P_{C} h w$ matrix that transforms $c$ full color image vector into $x$ mosaiced image vector by choosing selectively according to spatial arrangement of colors.

$$
\begin{align*}
& x=M c \\
& x=M A y \tag{2}
\end{align*}
$$

See Figure 3, for the visual representation of the same. The calculation of demosaicing matrix $D$ with LMMSE using sliding neighborhood ( $n_{h}, n_{w}$ ) has been described for RGB CFAs [5]. Here we will unfold each $x$ basis pattern plus neighborhood pixels using the constant neighborhood giving a size of $x_{1}$ equal to $\left(h+n_{h}-1\right)\left(w+n_{w}-1\right) \times H W /(h w)$ [4]. Similarly for $y$, we write $y_{1}$ which incorporates neighborhood. Now the computation of the demosaicing matrix from couples $\left(x_{1}, y\right)_{i}$ constructed from the database is given by the following equation:

$$
\begin{align*}
& \hat{y}=D x_{1} \\
& D=E_{i=1 . . k}\left\{\left(y x_{1}^{t}\right)\left(x_{1} x_{1}^{t}\right)^{-1}\right\} \tag{3}
\end{align*}
$$

We use this $D$ to calculate $\hat{y}$, the demosaiced image. It is possible to design a matrix $M_{1}$ having size $\left(h+n_{h}-1\right)\left(w+n_{w}-1\right) \times$ $P_{C}\left(h+n_{h}-1\right)\left(w+n_{w}-1\right)$ that transform a neighborhood in the color image vector $c$ into a neighborhood of the mosaiced image $x_{1}, x_{1}=M_{1} c_{1}$. Let $A_{1}$ be matrix A with neighborhood incorporated. $A_{1}$ is now of size $P_{C}\left(h+n_{h}-1\right)\left(w+n_{w}-1\right) \times P\left(h+n_{h}-\right.$ $1)\left(w+n_{w}-1\right)$. So $c_{1}$ can be expressed as $c_{1}=A_{1} y_{1}$. It is also possible to design a matrix $S_{1}$ that transform the vector $y_{1}$ into the vector $y, y=S_{1} y_{1}$, such that it suppresses the neighborhood and selects the central pattern. With these two matrices, $D$ can be expressed as:

$$
\begin{align*}
& x_{1}=M_{1} A_{1} y_{1} \text { and } y=S_{1} y_{1} \\
& D=E_{i=1 . . k}\left\{\left(S_{1} y_{1} y_{1}^{t} A_{1}^{t} M_{1}^{t}\right)\left(M_{1} A_{1} y_{1} y_{1}^{t} A_{1}^{t} M_{1}^{t}\right)^{-1}\right\} \\
& =\left(S_{1} R A_{1}^{t} M_{1}^{t}\right)\left(M_{1} A_{1} R A_{1}^{t} M_{1}^{t}\right)^{-1} \\
& \text { where } R=E_{i=1 . . k}\left\{y_{1} y_{1}^{t}\right\} \\
& k \text { is number of images in a database } \tag{4}
\end{align*}
$$

So starting from CFA image $x$ in the artificial color domain $C$ we can do demosaicing and directly recover a full RGB image. This


Figure 3. Matrix model of the multicolor CFA image formation without considering neighborhood
is an advantage of considering $C$ to be linear combination of RGB and not an arbitrary color.

## Finding Optimum CFA arrangement

Now for demosaicing any given CFA we need to generate its $S_{1}, M_{1}$, and $A_{1}$ matrices. We calculate $D$ as per equation 4 . We now need to multiply $D$ to the $x_{1}$ (unfolded CFA image) to get back the color reconstructed image. We use average $M S E$ as metric for estimation of image quality for a CFA across images of a database. Our model being linear we can express average $M S E$ as a trace of matrix multiplication as follows [26].

$$
\begin{aligned}
& \text { AverageMSE }=\frac{\sum M S E}{k} \\
& =\frac{\sum \sum(\hat{y}-y)^{2}}{k H W P}=\frac{\sum \sum\left(\hat{y}^{t}-\hat{y}^{t}-y y^{t}+y y^{t}\right)}{k H W P} \\
& =\sum \sum\left(D M_{1} A_{1} y_{1} y_{1}^{t} A_{1}^{t} M_{1}^{t} D^{t}-D M_{1} A_{1} y_{1} y_{1}^{t} S_{1}^{t}-\right. \\
& \left.S_{1} y_{1} y_{1}^{t} A_{1}^{t} M_{1}^{t} D^{t}+S_{1} y_{1} 1_{1}^{t} S_{1}^{t}\right) / k H W P
\end{aligned}
$$

replacing by R from equation 4
$=\operatorname{trace}\left(D M_{1} A_{1} R A_{1}^{t} M_{1}^{t} D^{t}-D M_{1} A_{1} R S_{1}^{t}-\right.$
$\left.S_{1} R A_{1}^{t} M_{1}^{t} D^{t}+S_{1} R S_{1}^{t}\right) / P h w$

The above term is independent of CFA image $x$ and gives us an indicator of performance directly from the cross correlation matrix $R$. Therefore by evaluating this equation once we directly compute the average $M S E$ and therefore it is very fast compared to averaging explicitly on $k$ images. We used the Matlabs fmincon function using active-set algorithm [13, 27, 28] to find the matrix $A$ which gives the minimum of average $M S E$ for a given basis pattern size.

## Results

We used the above methodology to find the optimum multicolor CFA (see Figure 4), CFAs labelled $h \times w_{m}$. We used our algorithm to test some state of the art multicolor CFAs. We also compared results of algorithm to those reported by other authors for their respective CFAs. The metrics we tested our algorithm for is average PSNR $\mu$, as

$$
\begin{equation*}
\text { Average PSNR } \mu=\frac{\sum P S N R}{k}=\left(\sum\left(10 \log _{10}\left(\frac{1}{M S E}\right)\right)\right) / k \tag{6}
\end{equation*}
$$

Note this is not the same as $10 \log _{10} \frac{1}{\operatorname{AvgMSE}}$ which can be evaluated through equation 5 . We use a neighborhood of 10 for evaluating our algorithm. We leave a border equal to neighborhood size, 10 here. Also we clip the images between [0 255]. For instance $\mu$ for Bayer CFA is 38.90 dB for unclipped and 39.13 dB clipped. We also tested color SSIM (average SSIM over RGB channel) [34], color difference $\Delta E$, variance of PSNR for individual RGB channel (lower value indicates all color channels are well reconstructed) $\sigma_{r g b}$ and variance of PSNR across all images in database $\sigma$. In the two tables the values we report are averages for all images in the database.

In the Table 1, the first subpart shows the comparison of the state of art CFAs with our LMMSE algorithm compared with the best state of art algorithms. Starting with Bayer, LLSC [21] has the best performance but it takes approximately 6.5 minutes to process a single image. ACUDE [38] is next but with the code available on their site it took approximately 1.6 hour to process a single image. Then we have algorithms like LPA-ICI [25] which at 40.52 dB compute in around 1 s . Although the performance of LMMSE is poor for Bayer CFA, it is considerably faster. For Hirakawa, Condat, Bai, Hao 4b CFAs our method outperforms others. Then for CFAs with white pixels like Hao40, Hao50, Hao60,


Figure 4. All CFAs. (a) Bayer, (b) RGBW, (c) Hirakawa [16], (d) Condat [9], (e) Bai [6], (f) Hao 4a, (g) Hao 4b [14], (h) Yamagami [37], (i) Kodak 2.0, (j) Sony RGBW, (k) Hao40, (I) Hao50, (m) Hao60 [38], (n) 2x2m, (o) $4 \times 4 m_{1}$, (p) $4 \times 4 m_{2}$, (q) 6x6m, (r) 8x8m, (m) 10x10m.

Table 1: LMMSE for Kodak database. Other represents the value from the best state of the art algorithms known to us. ${ }^{1}$ is LLSC [21]. ${ }^{2}$ is LS Condat. ${ }^{3}$ is Bai [6]. ${ }^{4}$ is ACUDE [38]. Refer to Figure 4 for the CFAs.

|  | LMMSE |  |  |  |  | Other |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CFA | $\mu$ | SSIM | $\Delta E$ | $\sigma_{r g b}$ | $\sigma$ | $\mu$ |
| bayer | 39.13 | 0.9913 | $\mathbf{1 . 4 0}$ | 4.85 | 6.22 | $\mathbf{4 1 . 4 6}^{1}$ |
| hirakawa | $\mathbf{4 0 . 4 5}$ | 0.9933 | 1.49 | 2.97 | 5.72 | $40.36^{2}$ |
| condat | $\mathbf{4 0 . 5 8}$ | 0.9938 | 1.49 | 1.18 | 6.23 | $40.11^{2}$ |
| bai | $\mathbf{4 0 . 7 7}$ | 0.9939 | 1.50 | 1.76 | 6.11 | $40.38^{3}$ |
| hao4b | $\mathbf{4 0 . 7 5}$ | 0.9938 | 1.52 | 1.47 | 5.78 | $40.73^{4}$ |
| hao4a | 40.49 | 0.9938 | 1.50 | 1.23 | 5.97 |  |
| kodak2.0 | 38.43 | 0.9902 | 1.80 | 2.21 | 5.84 | $\mathbf{3 8 . 7 0 ^ { 4 }}$ |
| sonyrgbw | 37.38 | 0.9882 | 1.95 | 3.54 | 5.66 | $\mathbf{3 8 . 1 0 ^ { 4 }}$ |
| hao40 | 38.66 | 0.9911 | 1.71 | $\mathbf{0 . 7 0}$ | 5.64 | $\mathbf{3 8 . 9 3 ^ { 4 }}$ |
| hao50 | 39.07 | 0.9917 | 1.69 | 2.23 | 5.86 | $\mathbf{4 0 . 6 1 ^ { 4 }}$ |
| hao60 | 37.45 | 0.9884 | 2.17 | 7.67 | 5.32 | $\mathbf{3 7 . 5 1 ^ { 4 }}$ |
| RGBW | 39.74 | 0.9926 | 1.59 | 1.89 | 5.69 |  |
| yamagami | 37.14 | 0.9874 | 1.99 | 3.96 | 5.93 |  |
| $2 \times 2 m$ | 40.08 | 0.9930 | 1.54 | 1.68 | 6.40 |  |
| 4×4m $m$ | 41.11 | $\mathbf{0 . 9 9 4 4}$ | $\mathbf{1 . 4 4}$ | $\mathbf{0 . 7 2}$ | 5.95 |  |
| $4 \times 4 m_{2}$ | $\mathbf{4 1 . 1 2}$ | 0.9943 | $\mathbf{1 . 4 4}$ | 0.81 | 5.94 |  |
| $6 \times 6 m$ | 41.09 | 0.9943 | $\mathbf{1 . 4 4}$ | 0.83 | 5.88 |  |
| $8 \times 8 m$ | 41.09 | 0.9943 | 1.46 | 0.76 | 5.86 |  |
| $10 \times 10 m$ | 40.51 | 0.9936 | 1.46 | 0.79 | 5.66 |  |

Sony RGBW and Kodak 2.0, ACUDE [38] is the best performer. We earlier mentioned the limitation of Bayer CFAs and requirement of computationally expensive algorithms to overcome that. Therefore we recommend multicolor CFA in the lower sub-part of above table which show the best CFAs we found for size 2 to 10 . We present two CFAs of size $4 \times 4$ to demonstrate that the CFAs found by our optimization process are not the best one but rather one of the several good performers. We are able to achieve a PSNR of 41.12 dB in less than 0.2 s which shows the efficiency of our implementation. In general LMMSE takes between 0.1 s to 0.6 s , depending on CFA size. Figure 5 shows the crop of the fencing region of the Lighthouse image from the Kodak database. For CFA size of $4 \times 4$ and higher it is color noise free.

Table 2 shows the results for our algorithm on the McM database [40]. Although we found ACUDE to have a better PSNR, yet it is computationally less efficient.

Table 2: LMMSE for McM database. In Other best results were from ACUDE [38].

|  | LMMSE |  |  |  |  | Other |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CFA | $\mu$ | SSIM | $\Delta E$ | $\sigma_{r g b}$ | $\sigma$ | $\mu$ |
| bayer | 35.70 | 0.9830 | 3.35 | 7.96 | 8.99 | $36.38^{4}$ |
| hirakawa | 35.22 | 0.9821 | 4.19 | 3.41 | 9.49 | $34.2^{4}$ |
| condat | 36.04 | 0.9851 | 3.99 | 0.94 | 9.23 | $35.42^{4}$ |
| bai | 35.24 | 0.9831 | 4.43 | 0.89 | 9.62 |  |
| hao4b | 35.63 | 0.9838 | 4.25 | $\mathbf{0 . 7 0}$ | 9.62 | $35.64^{4}$ |
| hao4a | 35.84 | 0.9845 | 4.14 | 1.36 | 9.13 |  |
| kodak2.0 | 34.74 | 0.9803 | 4.38 | 1.60 | 9.27 | $\mathbf{3 5 . 1 5 ^ { 4 }}$ |
| sonyrgbw | 34.46 | 0.9788 | 4.47 | 1.84 | 9.29 | $34.87^{4}$ |
| hao40 | 35.50 | 0.9832 | 3.96 | 1.40 | 9.06 | $36.21^{4}$ |
| hao50 | 35.72 | 0.9831 | 4.18 | 3.81 | 9.51 | $36.71^{4}$ |
| hao60 | 34.64 | 0.9796 | 4.83 | 7.00 | 9.58 | $35.31^{4}$ |
| RGBW | 35.86 | 0.9842 | 3.76 | 2.75 | 9.42 |  |
| yamagami | 34.55 | 0.9789 | 4.32 | 3.13 | 9.31 |  |
| $2 \times 2 m$ | 35.91 | 0.9845 | 3.77 | 4.63 | 9.47 |  |
| $4 \times 4 m_{1}$ | 35.90 | 0.9849 | 4.08 | 1.81 | 9.46 |  |
| $4 \times 4 m_{2}$ | 36.00 | 0.9852 | 4.03 | 2.27 | 9.40 |  |
| $6 \times 6 m$ | 35.71 | 0.9845 | 4.18 | 1.65 | 9.37 |  |
| 8×8m | 35.91 | 0.9849 | 4.14 | 1.86 | 9.27 |  |
| $10 \times 10 m$ | 35.64 | 0.9839 | 4.25 | 1.20 | 8.93 |  |

## DFTs of proposed CFAs

Figure 6, shows the average DFT response on the Kodak database, for the proposed CFAs. It can be seen that for CFAs proposed from size 4 to 10 , it won't be possible to use frequency selection method to separate luminance and chrominance.

## Discussion

Some of the proposed CFAs have something like a dark pixel, a pixel with a very low sensitivity. These dark pixels could lead to increase in noise. Actually with the LMMSE model the final value of a pixel depends not only on its own but also on its neighboring pixel and for these dark pixels the contribution of neighboring pixels compensates. We can mitigate the problem of noise by substituting dark pixels with normal ones. For instance for $2 \times 2 m$, the pixel $(1,2)$ is light blue having $\gamma$ of 0.357 . We can make it pure blue at 1 and we still get the same $\mu$. Similarly for $4 \times 4 m_{2}$, we have two dark pixels. We can make pixel $(2,2)$ green


Figure 5. Crop of Lighthouse image for proposed $2 \times 2 m, 4 \times 4 m_{1}, 4 \times 4 m_{2}$, $6 \times 6 \mathrm{~m}, 8 \times 8 \mathrm{~m}$, and $10 \times 10 \mathrm{~m}$ CFAs
and pixel $(4,4)$ as blue and we still have same $\mu$. The proposed CFAs are optimized ones, they are not necessarily the best ones. We start with a random CFA pattern and stop after a set number of iterations. We may continue the optimization process or choose a different random seed and get another random CFA which has equally good performance.

## Conclusion

In this paper we presented LMMSE with neighborhood approach for demosaicing CFAs, considering color filters as linear combination of RGB filters. Although it is not clear if these CFAs are physically realisable, this assumption has been used to propose optimal CFAs in the literature. We proposed a method to find the optimized color filter by expressing the average MSE as a function of the correlation matrix, demosaicing matrix and other parameters related to CFA patterns. We demonstrated our algorithm on CFAs with a white pixel also which should allow increased light sensitivity. The proposed algorithm has the best performance to computational complexity of those tested. The algorithm is generic and can be used for any random CFA. The proposed CFAs have performance higher than 41.1 dB which is amongst the best results in state of art. The proposed CFAs cannot be demosaiced by frequency selection method, therefore for random CFAs, LMMSE despite being linear is a good solution. We also found some optimized CFAs had something like a dark pixel. Despite that the neighborhood compensates the sub-sampling by the mosaic by adding redundancy and improves the color reconstruction.

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Figure 6. DFTs of proposed (a) $2 \times 2 m$, (b) $4 \times 4 m_{1}$, (c) $4 \times 4 m_{2}$, (d) $6 \times 6 \mathrm{~m}$, (e) $8 \times 8 \mathrm{~m}$, and (f) $10 \times 10 \mathrm{~m}$ CFAs
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