Neurogeometry of color vision

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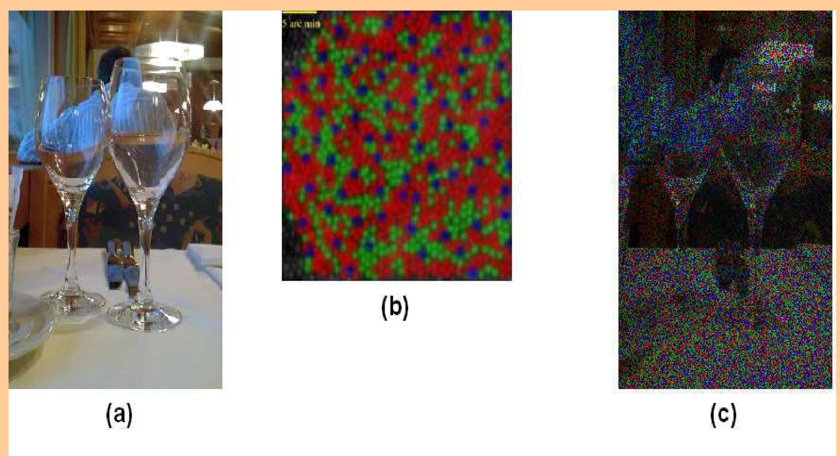
Introduction

Principles of differential geometry and neuron dynamics have been applied to visual perception of form, explaining illusory contours such as Kanisa's figure (see [1] for a review). Here we ask if such a framework could be applied to color vision. More generally, since in photopic vision, only cones are functionning in the retina we discuss how the achromatic information could be estimated from the chromatic samples of cones, thus allowing the application of this framework for spatial vision.

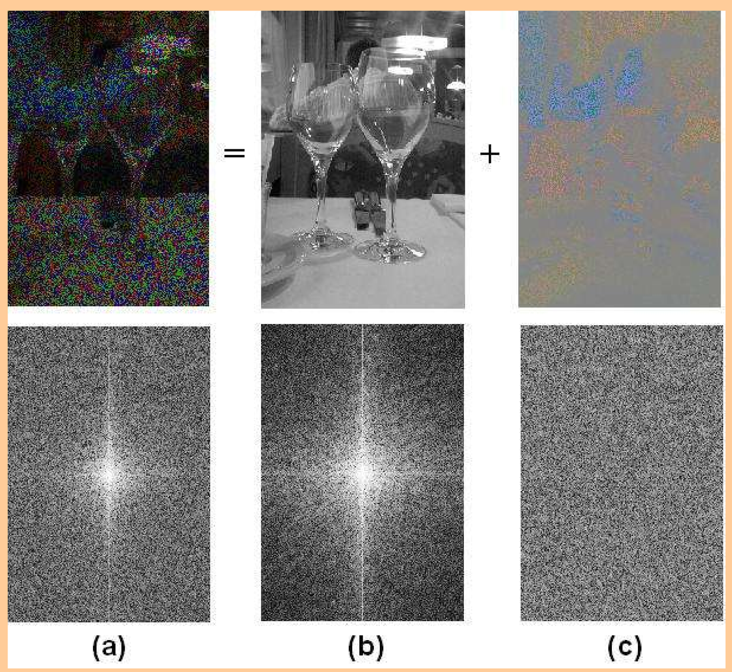
Color is a three dimensional space. We can think of applying neurogeometry to the three dimensional space of color vision by considering a RxC space (with R the retina domain and C the color domain). Actually, this three dimensional representation does not exist in the retina. Instead, the sampling of colors is made trough the chromatic mosaic of cones. Thus, color in the visual system correspond to a scalar array (similar to R) containing the three responses of the three different cones grouped in a single array.

We show how we can take into account the mosaic for neurogeometric model and also how the abundant literature on color discrimination defining a differential space could be used in neurogeometry to characterize the processing of colors by the visual system.

Illustration of the chromatic retinal image, only a single chromatic sensitivity is available per spatial position



Decomposition of a cone sampled image into its achromatic and chromatic components. The corresponding Fourier spectrum shows the random modulation due to the mosaic.



Is there any "wavelet" representation for color in the visual system?

Decomposition of a color image into its achromatic and chromatic components. Achromatic signal is defined as a line in the corresponding color space. This line is a subspace which could be considered as a linear variety.

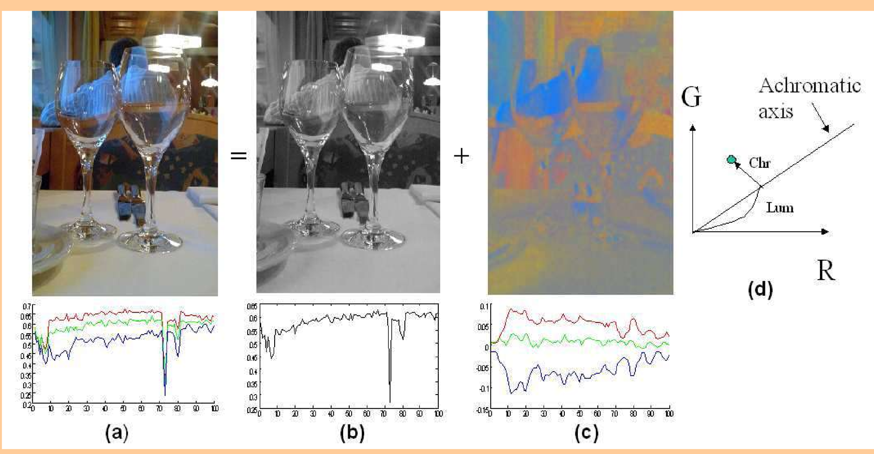
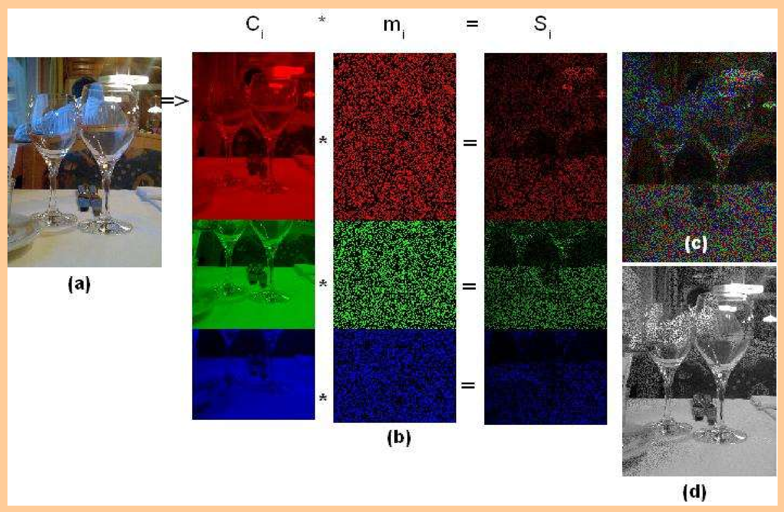
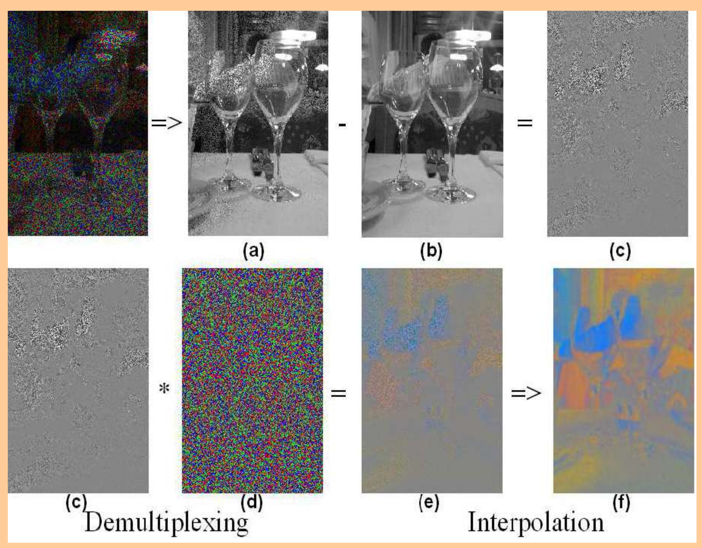


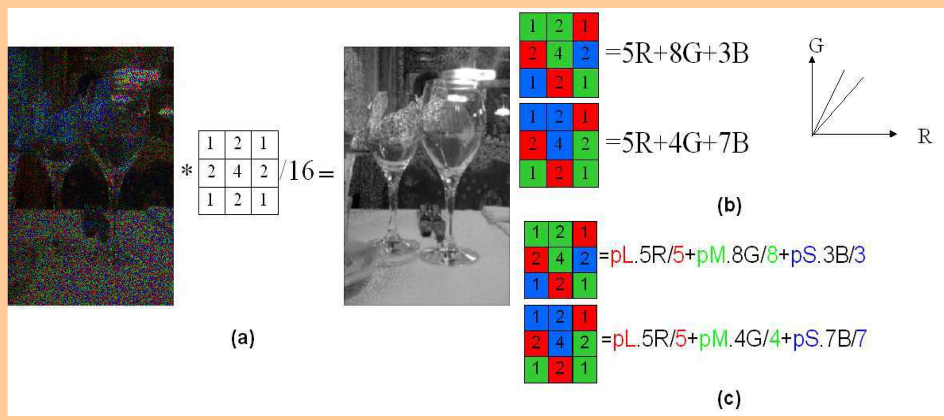
Illustration of the construction of the chromatic mosaic from a color image. The operation is a projection through the lattices defined by the arrangement of chromatic channel in the lattice.



Mosaic image could be seen as the sum of an achromatic image plus a subsampled and modulated chromatic image. This means that only the chromatic information is subsampled and modulated acceordingly to the mosaic arrangement. Achromatic information is fully defined



When applying a linear local operator to the mosaic for estimating achromatic information, the local arrangement of chromatic samples gives several definitions for the linear variety. With normalized operator it is no more the case.



One of the key mechanism for neurogeometry is the definition of a multiresolution local operator that allows the analysis of the content of the incoming visual image in term of orientation and chromatic content. In the perception of form, the operators used are either Gabor wavelets or more sophisticated wavelets specialized in analysing the geometry of image [2, 3].

But for cone image in the retina, the neighborhood is composed by different chromatic content from position to position along the retina because of the random arrangement of chromatic samples in the mosaic. This prevent to use operator design for form analysis. Particularly, in that case a local linear operator such as achromatic operator would have different definition following the local arrangement on the mosaic.

We can think of using a multiresolution systems dedicated to non regular sampling signals. But these systems are difficult to design [4]. Rather, we can think that the spatial and chromatic information is reconstructed as a RxC space before the analysis is performed. There is no evidence that such a reconstruction is happening in the visual system [5]. But if we find a way to reconstruct achromatic and chromatic information from the mosaic we can hopefully integrate it in a multiresolution local operator.

In [6] we show that a normalized convolution operator which takes the proportion of each color channel as a normalization factor is able to extract achromatic independently from chromatic information in a mosaic with random arrangement. Using this operator again for chromatic interpolation, we were able to reconstruct a full color image from its randomly arranged mosaic.

Even if the reconstruction of color is not perfect with such an operator (a formal analysis remains to be pursued) we hope to be able to integrated it in a multiresolution framework since its operator is still local. The remaining problem is that the arrangement in the mosaic should be known to allow a proper demultiplexing of chromatic information. At least the normalized convolution allows -the application- of form neurogeometry on the achromatic information extracted from the mosaic.

Is there any differential system for modelling dynamics of color perception?

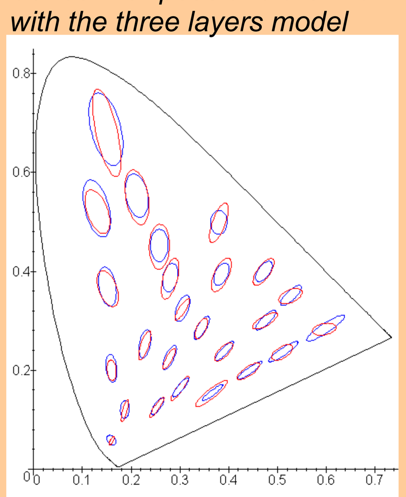
In color vision there is a huge litterature of color discrimination [7] that is used to discover the mechanisms of color perception. One of the pioneer of these studies is Macadam who found that in XYZ color space the threshold of just noticeable differences between colors are ellipsoids. This is illustrated by the well known MacAdam ellipses in the projective xy space.

Today we know that several factors such as the choice of color space, the spatial and temporal behavior of the stimuli influences the result of discrimination. But there is a framework that could be drawn to take into account most of the discrimination data. This framework is linear and comprise an adaptation level (usually Von Kries type) a coding level of response of cones into an achromatic and two chromatic channels followed by a desensitization level which is also adaptive.

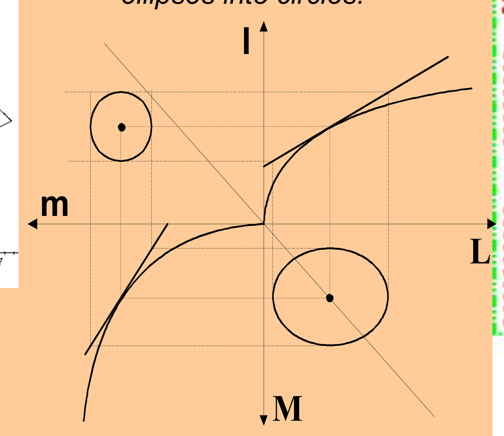
We have study [8] a model of this kind to take into account MacAdam ellipses and several other discrimination data transform into LMS space. This model is based on a adaptive non linearity call the Michaëlis-Mentens function. It is given by Y = X/(X+X0) where on the photoreceptor side, Y is the response of a photoreceptor, X is the transduction of light by photoreceptors, and X0 is the adaptation state. We show that this law allows explaining the behavior of Macadam ellipses supposing that through the three layers model, the threshold is estimated as a distance in Euclidean space. Thus $\Delta s = y'^{T}y'$ with $y=G_b(M.F_a(X))$, where X is the response of cones, F the adaptive non-linearity with parameter a, M the transformation from cone space to achromatic and chromatic components and G the desensitization with parameter b.

We used this three layer model as a local operator on image processing [9]. In this case the parameter X0 correspond to a convolution of the image with a local spatial filter. We apply this operator to the raw image directly coming for a digital camera before applying the color reconstruction. The result of tone mapping with this operator is better than the classical global gamma function used in image processing.

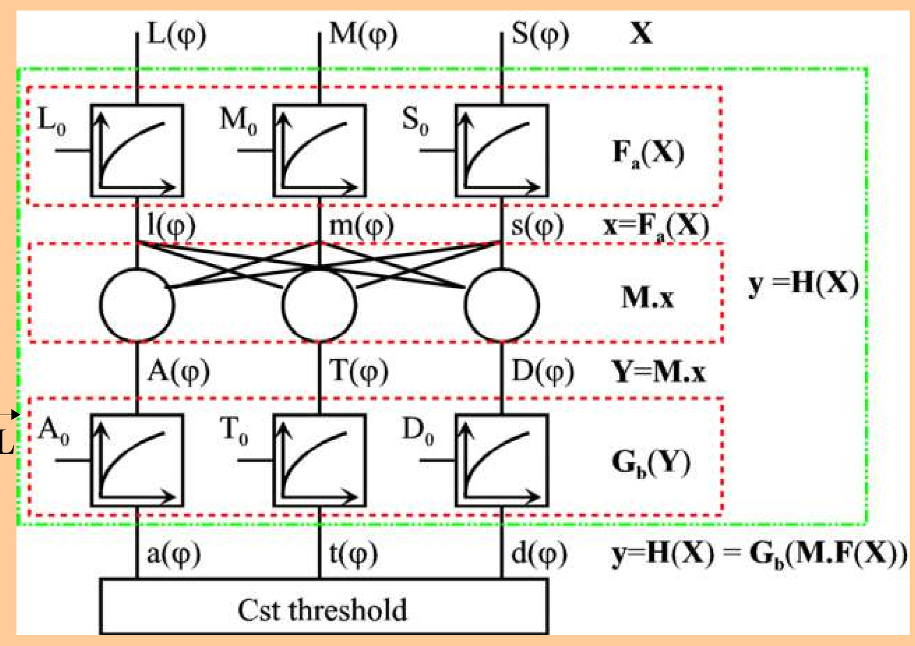
MacAdam ellipses and their fit with the three layers model



The linear approximation arround the center of the ellipse of a non linear law could transform the ellipse into a circle. If the non linear law is parametric with an adaptation parameter we may hope transforming a group of ellipses into circles.



The three layers model used to account for discrimination data



When applying the model as a local operator for image tone mapping the result is better than using the classical image processing

Conclusion

Neurogeometry does not apply directly to color vision because the sampling of colors through the random mosaic of cones implies a non trivial projection of the three dimensional color space to a scalar. Also this projection is not easily invertible. Moreover, the structure of the mosaic modifies the statistic of the sample neural information. Even if the physics of the visual scene is stationary in spatial and chromatic dimension, it does not remain true for responses of photoreceptor due to the mosaic. The second problem is that the dynamics of the color perception is not yet modeled completely. We are far from having a clear idea between the physiology of neuron in the visual system and the behavior of color perception such that we have between V1 and form perception.

Here we show that a normalized convolution kernel allows the identification of achromatic versus chromatic content of a mosaic image. We also show that a model of color dicrimination allows to draw a general schema of color perception that could apply well in image processing as a local operator. It remains to integrate these different parts into a complete neurogeometric framework for color perception.

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